

## String universality in ten dimensions

Allan Adams<sup>1</sup>, Oliver DeWolfe<sup>2</sup> and Washington Taylor<sup>1</sup>

<sup>1</sup> *Center for Theoretical Physics  
Massachusetts Institute of Technology  
Cambridge, MA 02139, USA*

<sup>2</sup> *Department of Physics 390 UCB  
University of Colorado  
Boulder, CO 80309, USA*

**Abstract**

We show that the  $\mathcal{N} = 1$  supergravity theories in ten dimensions with gauge groups  $U(1)^{496}$  and  $E_8 \times U(1)^{248}$  are not consistent quantum theories. Cancellation of anomalies cannot be made compatible with supersymmetry and abelian gauge invariance. Thus, in ten dimensions all supersymmetric theories of gravity without known inconsistencies are realized in string theory.

## Introduction

Supersymmetry and anomaly cancellation place strong constraints on quantum theories of gravity. Such constraints are strongest in higher dimensions. In eleven dimensions there is a unique theory of gravity compatible with supersymmetry. This theory is believed to be described as a UV-complete quantum theory by the branch of string theory known as “M-theory”. Similarly, in ten dimensions with two supersymmetries ( $\mathcal{N} = 2$ ), there are only two consistent supergravity theories, known as type IIA and IIB. Both of these are realized as limits of string theory. In these highly supersymmetric situations, then, we have “string universality,” meaning that all theories without known inconsistencies are realized in string theory.

As the dimension and number of supersymmetries decreases, the range of possible theories dramatically increases. In four space-time dimensions with one or no supersymmetries, we have only a limited understanding of the range of possible string compactifications, but our knowledge of consistency conditions needed for UV completion is still weaker (see for example [1]). The term “swampland” has been used to characterize the set of theories which cannot be ruled out from knowledge of the low-energy physics, and yet which cannot be realized in string theory [2]. Given our limited knowledge of both the space of string compactifications and constraints on consistent quantum gravity theories, the apparent swampland is a moving target, decreasing in scope whenever new string vacuum constructions or quantum consistency constraints are identified. Although the space of four dimensional theories has been so far hard to characterize globally, it was conjectured in [3] that  $\mathcal{N} = 1$  supergravity theories in six dimensions satisfy string universality. There are, however, some theories in this class which still lie in the apparent swampland, neither provably inconsistent nor as-yet realized in string theory [4], so this conjecture remains to be conclusively proven or disproven.

In the present work, we reconsider the simplest and most symmetric class of supergravity theories where string universality is in doubt, namely  $\mathcal{N} = 1$  supergravity in ten dimensions. Supersymmetry allows the addition of Yang-Mills fields in such theories, and cancellation of gravitational and gauge anomalies [5] requires the Green-Schwarz mechanism [6]. The only consistent choices of the gauge group  $G$  without abelian factors,  $G = SO(32)$  and  $G = E_8 \times E_8$ , are realized as the type I and heterotic limits of string theory. It has been noted that the gauge groups  $G = U(1)^{496}$  and  $G = E_8 \times U(1)^{248}$  satisfy some of the same conditions as the  $SO(32)$  and  $E_8 \times E_8$  cases [7], and therefore might appear to be consistent, anomaly-free theories. They are thus candidates for the swampland, since they have no known embedding into string theory. In [8] Fiol noted several properties of these theories, related to their moduli spaces and singularities under compactification, suggesting they cannot be embedded into a theory of quantum gravity.

In this note, we demonstrate that these theories with abelian gauge group factors indeed cannot be consistent supersymmetric quantum theories of gravity. In brief, although the anomaly factorizes as needed for the Green-Schwarz mechanism, the abelian gauge fields do not participate in hexagon diagrams, so there are no abelian anomalies to cancel. Supersymmetry, however, requires a coupling of the  $B$  field to the abelian gauge bosons of the form  $BF^2$ , just as in the non-abelian case. In using the  $B$  field to cancel the gravitational and non-abelian gauge anomalies, the standard Green-Schwarz mechanism generates tree diagrams with external abelian gauge bosons of the form in Figure 1 with no associated anomaly. This corresponds to a breakdown of gauge invariance for

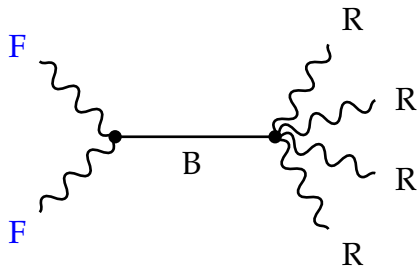


Figure 1: Green-Schwarz-type tree diagram arising in  $\mathcal{N} = 1$  supergravity theories in ten dimensions with abelian factors. With no corresponding anomaly to cancel, these theories are not gauge invariant

theories with abelian factors. As a result, we find that ten dimensional supergravity theories appear to manifest string universality — all consistent theories have string theory embeddings, and the 10D supergravity swampland is empty.

### Anomaly cancellation in non-abelian theories

We begin by reviewing Green-Schwarz anomaly cancellation for the purely non-abelian theories. The bosonic fields of ten-dimensional  $\mathcal{N} = 1$  supergravity plus super-Yang-Mills consist of a metric  $G_{MN}$ , the dilaton  $\Phi$ , a 2-form  $B_2$  and the gauge field  $A = A^a T^a$  with  $T^a$  the generators of the gauge group. The action for these fields can be written as

$$S = \frac{1}{2\kappa_{10}^2} \int e^{-2\Phi} \left( *R + 4d\Phi \wedge *d\Phi - \frac{1}{2} H_3 \wedge *H_3 - \frac{1}{2} F^a \wedge *F^a \right), \quad (1)$$

with  $*R = d^{10}x \sqrt{-G}R$ . This theory also includes three fermions, a gravitino  $\Psi_M$ , a dilatino  $\lambda$ , and a gaugino  $\chi \equiv \chi^a T^a$ , whose contributions to the action we suppress. It was shown by Bergshoeff et al. [9] for the case of an abelian gauge group, and extended to the non-abelian case by Chapline and Manton [10], that supersymmetry requires the field strength of the 2-form to acquire an extra piece consisting of the Chern-Simons term for the gauge group,

$$H_3 \equiv dB_2 - \omega_3^Y, \quad (2)$$

where

$$\omega_3^Y \equiv A^a \wedge dA^a + \frac{2}{3} f^{abc} A^a \wedge A^b \wedge A^c. \quad (3)$$

As a result, invariance under the gauge group is only maintained if  $B$  transforms nontrivially under this group as well. If

$$A \rightarrow A + d\Lambda - i[A, \Lambda], \quad B_2 \rightarrow B_2 + \text{Tr}(\Lambda F), \quad (4)$$

with  $\Lambda$  an algebra-valued 0-form, then  $H_3$  is gauge invariant.

As is well-known, possible gauge groups for this theory are highly constrained by anomaly cancellation. For theories in ten dimensions, the anomaly is conveniently expressed in terms of a formal twelve-form anomaly polynomial  $\hat{I}_{12}(F, R)$ , constructed out of the gauge and tangent bundle curvature 2-forms. This obeys the descent relations,

$$\hat{I}_{12} = d\hat{I}_{11}, \quad \delta\hat{I}_{11} = d\hat{I}_{10}, \quad (5)$$

where  $\delta$  denotes the combined gauge and local Lorentz transformation. The failure of the path integral to be gauge invariant is then given by the integral of  $\hat{I}_{10}$ ,

$$\delta \log Z \sim \int d^{10}x \hat{I}_{10}. \quad (6)$$

The fermions are all Majorana-Weyl fields, and thus being chiral they contribute to both gauge anomalies (the gaugino) and gravitational anomalies (gaugino, dilatino and gravitino). These contributions come from hexagon diagrams with six external gauge bosons and/or gravitons coupled to the various fermi fields running in a loop. The contributions to the anomaly polynomial from the fermions for a general non-abelian gauge group were computed by Alvarez-Gaumé and Witten [5], and, following the presentation of Polchinski [11], can be arranged to the form

$$\hat{I}_{12} = \frac{1}{1440} \left( -\text{Tr}F^6 + \frac{1}{48}\text{Tr}F^4\text{Tr}F^2 - \frac{(\text{Tr}F^2)^3}{14400} \right) + (n - 496) \frac{\text{tr}R^6}{725760} - \frac{Y_4 X_8}{384}, \quad (7)$$

where  $\text{Tr}$  is the trace in the adjoint representation of the gauge group (supersymmetry requires that the gauginos running in the loop transform in the adjoint),  $\text{tr}$  is the trace in the fundamental of  $SO(1, 9)$ ,  $n$  is the total number of gauge bosons and

$$Y_4 = \text{tr}R^2 - \text{Tr}F^2, \quad X_8 = \text{tr}R^4 + \frac{(\text{tr}R^2)^2}{4} - \frac{(\text{Tr}F^2)(\text{tr}R^2)}{30} + \frac{\text{Tr}F^4}{3} - \frac{(\text{Tr}F^2)^2}{900}. \quad (8)$$

The Green-Schwarz anomaly cancellation mechanism is possible when the first two terms in (7) vanish, so that the anomaly takes the factorized form of the final term. For gauge groups  $SO(32)$  and  $E_8 \times E_8$ , the term inside large parentheses in (7) vanishes due to particular identities of those groups, and  $n = 496$  for both, killing the middle term.

For the remaining  $Y_4 X_8$  term to be cancelled through the Green-Schwarz mechanism, the three-form field strength must be enhanced at higher orders in the derivative expansion by a Chern-Simons term in the spin connection,

$$H_3 \equiv dB_2 - \omega_3^Y + \omega_3^R, \quad (9)$$

and the action must include at higher order the Green-Schwarz term,

$$\Delta S \sim \int B_2 \wedge X_8. \quad (10)$$

To preserve local Lorentz invariance the two-form  $B_2$  must now transform as

$$B_2 \rightarrow B_2 + \text{Tr}(\Lambda F) - \text{tr}(\Theta R). \quad (11)$$

Due to this modified transformation, the Green-Schwarz term is not gauge invariant; instead, because

$$Y_4 = d(\omega_3^R - \omega_3^Y), \quad \delta(\omega_3^R - \omega_3^Y) = d\delta B_2, \quad (12)$$

we have from the descent relations that the gauge variation of (10) is exactly of the correct form to cancel the anomaly in (6). Diagrammatically, a tree diagram with a  $B$  propagator connecting  $BY_4$  and  $BX_8$  vertices is generated by the cross-terms in  $|H_3|^2$  induced by (9) and the Green-Schwarz term, cancelling the anomaly from the hexagon diagrams. Notice that the first equation in (12) is equivalent to

$$Y_4 = dH_3. \quad (13)$$

This coincidence between the factorized anomaly polynomial (7) and the modified field strength required by supersymmetry (9) is the essential relation that allows the Green-Schwarz mechanism to operate.

Through this mechanism, the non-abelian  $SO(32)$  and  $E_8 \times E_8$  supergravities can be free of quantum anomalies. Indeed, since these supergravities are the low-energy limit of the type I and heterotic string theories, where the higher-derivative terms in (9) and (10) arise automatically, these theories seem to be consistent quantum theories in the UV.

### 10D supergravity with abelian gauge group factors

We now turn to the primary focus of this note, the theories with gauge groups  $G = U(1)^{496}$  and  $G = E_8 \times U(1)^{248}$ . These theories both contain  $n = 496$  generators, and the term in large parentheses in (7) vanishes for both, so they again have anomaly polynomials which take the factorized form  $Y_4 X_8$  [7]. Naively it then appears that the Green-Schwarz mechanism can again be brought to bear. We argue that this is not the case. The crux of the issue is that the Green-Schwarz term needed to cancel gravitational anomalies is required by supersymmetry to have a non-trivial abelian gauge variation, but there is no abelian anomaly to cancel this variation.

To see this, recall that the  $F$ -dependent terms in the anomaly polynomial (7) were generated by loops of gauginos coupling to external gauge bosons. Since, however, the general coupling between a gaugino and the gauge fields is

$$D\chi^a = \partial\chi^a - if^{abc}A^b\chi^c, \quad (14)$$

a  $U(1)$  gauge field, for which all associated structure constants vanish, decouples from the gauginos and does not appear in hexagon diagrams at all. This means that the anomaly polynomial should be independent of all abelian field strengths.

This is in fact already encoded in (7), since  $\text{Tr}$  is the trace in the adjoint, and the adjoint of  $U(1)$  is the singlet, so these traces vanish. Indeed this is why the term in parentheses vanishes for these theories; for  $U(1)^{496}$  it is simply zero term by term, and for  $E_8 \times U(1)^{248}$  all the abelian generators drop out analogously, while  $E_8$  alone obeys the same relation used in the  $E_8 \times E_8$  case. Correspondingly,  $Y_4$  and  $X_8$  in these cases lose all the traces over the  $U(1)$  generators. For  $G = U(1)^{496}$  this is particularly simple,

$$Y_4^{(496)} = \text{tr} R^2, \quad X_8^{(496)} = \text{tr} R^4 + \frac{(\text{tr} R^2)^2}{4}, \quad (15)$$

while for  $G = E_8 \times U(1)^{248}$  they take the form (8) but with  $E_8$  field strengths only. The only anomalies that must be cancelled, then, are gravitational and non-abelian gauge anomalies.

As shown by Bergshoeff et al. [9], however, supersymmetry requires that each abelian factor  $U(1)^{(i)}$  contributes an abelian Chern-Simons contribution to  $H_3$  of the form

$$H_3 \equiv dB_2 - \sum_i \omega_3^{(i)} + \dots, \quad (16)$$

where  $\omega_3^{(i)} = A^{(i)} \wedge dA^{(i)}$  and  $\dots$  indicates gravitational and possible non-abelian Chern-Simons contributions. As a result, the kinetic term for  $B_2$  is only gauge-invariant if  $B_2$  transforms under a general abelian gauge transformation as

$$\delta_\Lambda B_2 = \sum_i \Lambda^{(i)} F^{(i)}. \quad (17)$$

To cancel the gravitational and possible non-abelian anomalies, we need a Green-Schwarz term of the usual form (10). The Green-Schwarz term is not invariant under the abelian factors in the gauge group, however, transforming as

$$\delta_\Lambda \int B_2 \wedge X_8 = \int \sum_i \Lambda^{(i)} F^{(i)} \wedge X_8. \quad (18)$$

Since there is no abelian anomaly to cancel this gauge variation, the abelian symmetries are explicitly violated. Alternately, preserving gauge invariance under the abelian factors forbids this Green-Schwarz term, whose absence would leave an uncanceled local Lorentz anomaly. Supersymmetry thus allows us to preserve either abelian gauge or local Lorentz invariance, but not both.

The inapplicability of the Green-Schwarz mechanism in these cases can be viewed as the failure of the abelian theories to satisfy (13),

$$Y_4 \neq dH_3. \quad (19)$$

The abelian fields decouple from hexagon diagrams and hence drop out of  $Y_4$ , but are kept in  $dH_3$  by the demands of supersymmetry. This leads to tree-level contributions of the form  $F^2 R^4$  to the anomaly polynomial which do not correspond to any one-loop anomalies; correspondingly, the descent relations no longer imply that the variation of  $B_2$  can cancel the complete anomaly. Thus the ten-dimensional  $\mathcal{N} = 1$   $U(1)^{496}$  and  $E_8 \times U(1)^{248}$  theories cannot be made anomaly-free by the Green-Schwarz mechanism. We have thus shown that there are no consistent supergravity theories in ten dimensions that cannot be obtained from string theory.

## Acknowledgments

We would like to thank Dan Freedman, Michael Green, Vijay Kumar, Joe Minahan, Daniel Park, Joe Polchinski and John Schwarz for helpful discussions and correspondence. O.D. would like to thank the Center for Theoretical Physics at MIT for hospitality. We would like to particularly thank the Goosebeary's lunch truck where this work was completed. The research of A.A. and W.T. was supported by the DOE under contract #DE-FC02-94ER40818. The research of O.D. was supported by the DOE under contract #DE-FG02-91-ER-40672.

## References

- [1] A. Adams, N. Arkani-Hamed, S. Dubovsky, A. Nicolis and R. Rattazzi, “Causality, analyticity and an IR obstruction to UV completion,” JHEP **0610**, 014 (2006) [arXiv:hep-th/0602178](#).
- [2] C. Vafa, “The String landscape and the swampland,” [hep-th/0509212].  
N. Arkani-Hamed, L. Motl, A. Nicolis and C. Vafa, “The string landscape, black holes and gravity as the weakest force,” JHEP **0706**, 060 (2007) [arXiv:hep-th/0601001](#).  
H. Ooguri and C. Vafa, “On the geometry of the string landscape and the swampland,” Nucl. Phys. B **766**, 21 (2007) [arXiv:hep-th/0605264](#).
- [3] V. Kumar and W. Taylor, “String Universality in Six Dimensions,” [arXiv:0906.0987 \[hep-th\]](#).
- [4] V. Kumar and W. Taylor, “A bound on 6D  $\mathcal{N} = 1$  supergravities,” JHEP **0912**, 050 (2009) [arXiv:0910.1586 \[hep-th\]](#).  
D. R. Morrison, V. Kumar and W. Taylor, “Mapping 6D  $\mathcal{N} = 1$  supergravity to F-theory,” [arXiv:0911.3393 \[hep-th\]](#).  
D. R. Morrison, V. Kumar and W. Taylor, “Global aspects of the space of 6D  $\mathcal{N} = 1$  supergravities,” *to appear*.
- [5] L. Alvarez-Gaumé, E. Witten, “Gravitational Anomalies,” Nucl. Phys. **B234**, 269 (1984).
- [6] M. B. Green, J. H. Schwarz, “Anomaly Cancellations in Supersymmetric D=10 Gauge Theory and Superstring Theory,” Phys. Lett. **B149**, 117-122 (1984).
- [7] M. B. Green, J. H. Schwarz and E. Witten, “Superstring Theory. Vol. 2: Loop Amplitudes, Anomalies And Phenomenology,” *Cambridge Univ. Pr.* (1987)
- [8] B. Fiol, “Populating the swampland: The Case of  $U(1)^{496}$  and  $E_8 \times U(1)^{248}$ ,” [arXiv:0809.1525 [hep-th]].
- [9] E. Bergshoeff, M. de Roo, B. de Wit *et al.*, “Ten-Dimensional Maxwell-Einstein Supergravity, Its Currents, and the Issue of Its Auxiliary Fields,” Nucl. Phys. **B195**, 97-136 (1982).
- [10] G. F. Chapline, N. S. Manton, “Unification of Yang-Mills Theory and Supergravity in Ten-Dimensions,” Phys. Lett. **B120**, 105-109 (1983).
- [11] J. Polchinski, “String theory. Vol. 2: Superstring theory and beyond,” *Cambridge Univ. Pr.* (1998)